PHOTONS AND ELECTRONS IN MESOSCOPIC STRUCTURES: USEFUL ANALOGIES IN APPLICATION TO OPTICAL COMMUNICATION

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Abstract

In the context of the existing similarities in electron and optical properties of mesoscopic structures the problem is discussed of engineering the heterogeneous media and devices with predetermined optical transmission spectrum. Taking into account that similarities come from isomorphism of Shroedinger equation for an electron and Helmholz equation for electromagnetic waves in an inhomogeneous medium, a possibility is considered to transfer experience in solving inverse Shroedinger problem in quantum physics to photonic engineering where inverse Helmholtz problem is to be dealt with.

1 Introduction

Since the time Louie de Broglie had published his genuine hypothesis on wavy properties of material particles[1], the wave mechanics of matter has become a well developed field of modern science providing explanations of basic properties of the natural matter, e.g. atoms, molecules, clusters, crystals, and predicting new properties of artificial structures, e. g. quantum wells and superlattices. At the dawn of quantum mechanics, a transfer of ideas from classical physics of waves, namely diffraction and interference of electromagnetic (EM) waves, have been successfully performed interpret and predict the relevant phenomena for electrons. Nowadays in many instances one can observe a reverse process: concepts from condensed matter physics relevant to one-electron problems are being extensively transferred to optics and basically to physics of classical waves in a more general meaning.

Quantum particles of matter, e. g. electrons possess non-zero rest mass, charge, spin, and Fermi-Dirac statistic. Photons have zero rest mass, zero charge, no spin and obey Bose-Einstein statistic. Electrons and EM-waves have different densities of states and different dispersion laws. These characteristics are shown in Table 1. One can see these differences are rather significant resulting in different properties of matter and field. Nevertheless, there is a wide class of physical problems and phenomena where wave-like particles of matter and field quanta behave alike. These are problems and phenomena where Coulomb interactions, many-body effects and spin effects with respect to electrons, and absorption/emission events with respect to photons are not important. These problems are related to energy spectra of electrons in various potentials and to light propagation through different heterogeneous media. In many instances, electrons and classical waves behave similar if potential relief with respect to electrons is considered by analogy with local dielectric function with respect to EM-waves and if the relevant length scale is considered (nanometer scale in case of electrons and submicron scale in case of photons).

2 Isomorphism of Helmholtz and Schroedinger equations

Propagation of EM-waves and steady states of an electron are derived by means of solution of the same second order differential equation which is known as Helmholtz equation with respect to electromagnetic waves and Schroedinger equation[2] with respect to electron. It reads

$$\nabla^2 f(x,y,z) + F(x,y,z) f(x,y,z) = F_n f(x,y,z) . \qquad (1)$$

In case of an electron (or other quantum particle) *f(x,y,z)* is wavefunction which determines the probability to find a particle at a point *(x,y,z)*[3], *F(x,y,z)* determines the spatial potential relief *U(x,y,z)*, and F_n forms a set of values proportional to steady state particle energies. In case of EM-waves, *f(x,y,z)* is electric field spatial distribution, *F(x,y,z)* determines the spatial relief of dielectric function (in non-absorbing media the relief of refraction index *n(x,y,z)*), and F_n forms a set of propagating modes with frequencies ω_n. Fig.1 illustrates propagation of a particle or EM-wave through a heterogeneous medium with the relevant potential and refraction index profiles. For simplicity, a one-dimensional configuration with stepwise change of the corresponding physical parameter is presented which is the case for solid state heterostructures and optical multilayer filters for electrons and EM-waves, respectively.

Table 1. A sketch of principal characteristics of electrons and photons.
E – energy, *w* – frequency, *k* – wavenumber, *c* – speed of light in vacuum

	electrons	photons
rest mass	$m_0 = 9{,}109534 \cdot 10^{-31}$ kg	zero
charge	$å = 1{,}602189 \cdot 10^{-19}$ C	zero
spin	$^1/_2$	1
dispersion law (in free space)	$E = \mathbf{h}^2 k^2 / 2m_0$	$\mathbf{h}\omega = ck$
density of states (in 3 dimensions)	$D(E) = \dfrac{m^{3/2} E^{1/2}}{2^{1/2} \pi^{1/2} \mathbf{h}^3}$	$D(\omega) = \omega^2 / 2\pi^2 c^3$
statistic	Fermi–Dirac	Bose–Einstein

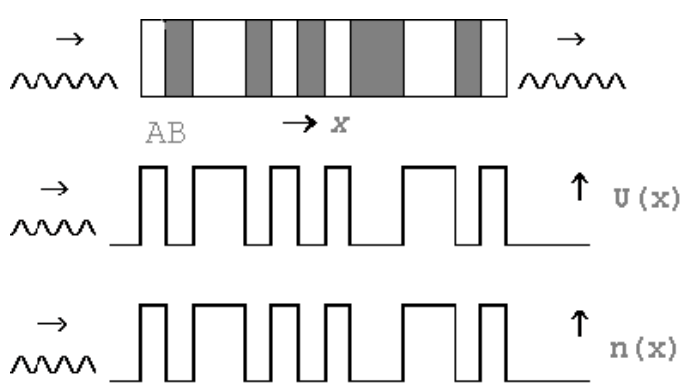

Fig. 1. A stack of alternating layers of two materials A and B with different parameters determining stepwise profile of potential *U(x)* and refraction index *n(x)*.

3 EM-waves and electrons in various media

There is a number of well-established analogies in behavior of electrons and waves in structures with spatial variation of potential and refractive index (dielectric function in more general case). They are listed in Table 2. Consider them very briefly using notation "potential" both for *U* and *n* profiles.

1. In case of a single potential step, reflection/transmission of a quantum particle in case of propagation over barrier is well established from a textbook solution of the Schroedinger

equation with a nice classical analogy in the form of Frenel's laws for reflection of light at the border of two media with different refractive indices. Note, the construction of non-reflecting barriers in quantum mechanics did stimulate development of optical thin film bleaching coatings [4].

2. Tunneling through a step-like potential barrier. This famous quantum effect first predicted by Leontovich and Mandelshtam[5] has a very clear classical analog, namely tunneling of light wave through thin metal films.

3. In case of sequential identical wells separated by identical barriers, splitting occurs of resonant states relevant to a single well which is known in textbook quantum mechanics and is inherent in multilayered coating widely used in optics and lasers as dielectric mirrors and beamsplitters[6].

4. Resonant tunneling of an electron through a couple of barriers separated by a well is widely used in nanoelectronics. The relevant classical analog is formation of resonant transmission modes in a Fabry-Perot cavity.

5. In a periodic potential, wavefunction and electric field distribution obey so called Bloch waves[7]. Both electrons and EM-waves possess bands of allowed and forbidden states where allowed states correspond to transmission modes and forbidden states within gaps form reflection modes (stop-bands). The band-like structure was first obtained by Kronig and Penney just for a periodic square potential [8]. The concepts of Bloch waves and energy band structures forms the basis of quantum theory of solids. Replication of these concepts to EM-waves resulted in a rapid development of the concept of photonic crystals, i.e. media with periodic change of dielectric function in 1, 2, or 3 dimensions, with multiple applications in optoelectronics and optical communication [9-12].

Table 2. Analogies of electron and EM-wave with respect to propagation in a medium with spatial profile of potential and refraction index

	Spatial profile	electrons	EM-waves
1.	single step	reflection/transmission	reflection/transmission
2.	step-like barrier	tunneling	tunneling
3.	identical sequential barriers/wells	multiple splitting of steady state energy levels	multiple resonance transmission bands
4.	a well isolated with 2 barriers	resonant tunneling	Fabry-Perot resonant modes
5.	periodic	energy bands separated by gaps	frequency transmission bands separated by gaps
6.	random with small disorder	weak localization	coherent back-scattering
7.	random with strong disorder	localization	localization
8.	quasiperiodic	fractal Cantor-like set of energy level in quasicrystals	fractal Cantor-like transmission spectrum of Fibonacci filters

6. Propagation of an electron and EM-waves in a medium containing randomly distributed scatterers is characterized by a peculiar phenomenon known as weak localization of electrons due to quantum interference effects[13,14] and coherent back scattering in optics [15,16].

7. In case of a dense disordered medium localization of electrons occurs[17] providing a background of our understanding of electronic properties of amorphous solids. Replication of these ideas to EM-waves[18,19] did stimulate challenging experiments on light localization [20].

8. In nonperiodic but deterministic multilayer structures pronounced regularities have been predicted and observed as well. With respect to electrons in quasiperiodic potential energy spectrum in the form of a fractal Cantor set have been established [21]. It is the case of naturally existing quasicrystals. The optical analog is Fibonacci quasiperiodic stack of alternating layers which was shown to possess Cantor-like spectrum with triple self-similar transmission bands within a stop band[22].

4 Band engineering in nanoelectronics and photonic engineering in optics

A transfer of ideas and concepts from solid state physics to optics have led to rapid development of interesting solutions in photonic engineering during last decades. Among them, e.g. are manipulation with light propagation, control of bandwidths via localization, hollow waveguides[23], optical diode[24] and omnidirectional dielectric mirror[25]. In the nearest future one can foresee evaluation of scaling properties inherent in nonperiodic fractal stacks like, e.g. Cantor set[26]. This topic remains unexamined both with respect to electronics and optics. However, it can be very useful in the problem of engineering of filters and heterostructures with predetermined transmission spectrum.

In the context of the existing similarities in electron and optical properties of mesoscopic structures the problems of engineering the heterogeneous media and devices with predetermined optical transmission spectrum can be solved based on experience gained in quantum physics and nanoelectronics. Results obtained for Schroedinger equation with various potential profiles, for example, U-like, V-like, W-like, parabolic and more complicate (discussed e.g. in [27–30]) can be directly replicated to optical filters with the proper account for different $E(k)$ and $\omega(k)$ functions for electrons and EM-waves, respectively (Table 1). Possibly this can result in development of thin film filters with desirable transmission bands for optical communication (add/drop and wavelength division multiplexing components) which can be competitive with existing fiber Bragg gratings.

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