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Propagation of waves in layered structures viewed as number recognition

S.V. Gaponenko [a,*], S.V. Zhukovsky [a,b], A.V. Lavrinenko [b], K.S. Sandomirskii [a,*]

[a] *Institute of Molecular and Atomic Physics, National Academy of Sciences of Belarus, F. Skaryna Pr. 70, Minsk 220072, Belarus*
[b] *Physics Department, Belarussian State University, Minsk 220080, Belarus*

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Abstract

We propose to consider multilayer spatial structures as numbers. An arbitrary finite sequence of layers with N values of a material parameter which determines the speed of wave propagation is considered as a number written in the numeration system with base N. Within the framework of this approach propagation of classical waves and quantum particles can be treated as number recognition. A problem is formulated of identification of a type among spatial sequences featuring unique spectral portraits versus spatial structure. It is shown possible to perform certain arithmetic operations by means of sequential propagation of waves through several structures. Using fractal Cantor structures as a representative example, spectral properties of waves are shown to reproduce certain properties of the corresponding numbers. A possibility is outlined to use the above approach for data storage. If a set of numbers possessing unique spectral portraits forms a complete set, then compact coding of arbitrary numbers will become possible. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Propagation of waves in heterogeneous media which have topological inhomogeneities along propagation direction with size comparable to wavelength in vacuum or in a continuous medium is characterized by multiple scattering and interference. In a medium with randomly distributed spatial inhomogeneities, wave propagation is diffusive and in many cases wave localization occurs. In deterministic non-continuous media, resonance propagation (tunneling) of wave is possible. If inelastic scattering (dissipation) of waves in a medium is negligible, propagation features complementary spectra of resonant transmission and reflection. These are determined by the spatial distribution of the material parameter which determines speed of the wave propagation. These parameters are potential energy for quantum particles, refraction index for electromagnetic waves, material density for acoustic waves. Multilayered media with stepwise variation of the parameter determining wave

* Corresponding author. Tel.: +375-172-684-750; fax: +375-172-393-131.

E-mail addresses: gaponen@imaph.bas-net.by (S.V. Gaponenko), lavrinenko@phys.bsu.unibel.by (A.V. Lavrinenko).

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propagation velocity comprise a specific case of deterministic non-continuous media.

In the present contribution, we propose to consider multilayered spatial structures as numbers. Within the framework of such consideration, *propagation* of classical waves and quantum particles can be treated as *number recognition*. The problem is posed of determination of a group of sequences which possess a single-value relationship of the spatial structure (i.e., number value) and resonant frequency (energy) spectrum. The proposed consideration can be used for dense data storage.

The paper is organized as follows. In Section 2 we introduce a general problem formulation of considering an arbitrary multilayer structure as a number with discussion of its application for quantum and classical waves; in Section 3 we consider a specific case of structures built of two different materials which comprise binary numbers; in Section 4 we demonstrate a possibility to perform certain arithmetical operations by means of sequential propagation of waves through a couple of structures; in Section 5 a problem is posed of searching for at least one complete set of numbers possessing unique spectral portraits; in Section 6 we demonstrate that classes of numbers exist for which a definite correlation occurs between spectral portraits and number of prime factors. Finally, in Section 7 we summarize the principal results of the paper.

2. Problem formulation

Multiple scattering and interference of scattered waves can result in resonant transmission of waves in an inhomogeneous medium if a physical parameter which determines wave velocity experiences spatial variation along propagation direction featuring certain regularities. The representative examples can be found in quantum as well as in classical physics. In quantum physics (and in electronics) these are resonant tunneling of a particle through a sequence of identical potential barriers, non-reflecting motion of a particle over a set of identical wells or barriers, development of energy bands in periodic potentials (crystals and quantum-size superlattices), development of a fractal-like spectrum of a particle in a quasiperiodic potential (quasicrystals). In classical physics, similar phenomena can be recognized as well. In particular they are clearly identified for electromagnetic waves. These are development of resonant transmission bands in a Fabry–Perot interferometer, formation of stop bands and transmission bands in multilayered dielectric mirrors, fractal transmission spectra known for quasiperiodic multilayer structures. Mathematical isomorphism of the single-particle steady-state Schroedinger equation relevant to a particle in potential field and Helmholtz equation for waves in a medium with space-dependent dielectric function leads to the analogy of classical (e.g., electromagnetic) waves and quantum particles (e.g., electrons) in deterministic media. In quantum physics, many problems related to electron spectra in various deterministic inhomogeneous media were investigated earlier than their classical analogs. This happened because quantum-mechanical formulation of these problems arose in the course of the systematic analysis of electron properties of solids including crystals, quasicrystals and nanostructures. Later on quantum mechanical problems related to electron properties of heterogeneous media were posed by development of nanoelectronics which basically uses the ideas of quantum engineering. In the recent years, a transfer occurs of ideas, concepts and results from quantum to classical physics. It is favored by development of optics of complex media and nanostructures including photonic crystals, quasicrystals and other complex media with deterministic dependence of dielectric function versus coordinates.

A layered structure features a stepwise dependence of dielectric function or potential energy along the direction normal to the layer plane. These structures form a specific class of deterministic complex media. We propose to consider a structure containing N substances with different material parameters as a number written in base N. Then propagation of classical waves and quantum particles through such a structure can be viewed as number recognition. Layered structures built of *two* substances with different values of a physical parameter which determines wave propagation velocity will correspond to *binary* numbers. For example, we ascribe for clarity "1" to a layer with higher value of potential energy U in the case of quantum particles, refraction index n in the case of electromagnetic waves, density in the case of acoustic waves and "0" to a layer with the lower value of the proper parameter (Fig. 1). Then a finite sequence of layers can be treated as the relevant number. This case will be considered in what follows because of its simplicity and importance of binary numbers in modern informatics. For clarity, we shall consider electromagnetic waves and use optical terminology in discussions. By solving Schroedinger and Helmholtz equations with stepwise potential and relevant boundary conditions one can get spectrum of resonant transmission and reflection of quantum particles and electromagnetic waves for every specific structure. In a sense, solving of these equations gives a spectral portrait, which is characteristic for a given structure. All examples given in the following sections were obtained by means of numerical solutions of Helmholtz equation which is isomorphic to the single-particle steady-state Schroedinger equation. Therefore, all conclusions are equally valid for multilayered quantum-size structures with respect to resonant tunneling and propagation of electrons over barriers (wells). For the sake of simplicity and clarity we consider layers with equal optical thicknesses, i.e., those satisfying a condition $d_1 n_1 = d_0 n_0$, where d_i is the geometrical thickness and n_i is the refraction index of a given layer.

3. Identification of binary numbers

We start consideration with binary numbers consisting of alternating "0" and "1", i.e., 0, 1, 10, 101, 1010, 10101, 101010, 1010101, 10101010,... These correspond to decimal numbers 0, 1, 2, 5, 10, 21, 42, 85, 170, 341,... Every next term in this raw can be obtained either by multiplication of the previous one by 2 (if one write "0" in the end) or by multiplication of the previous one by 2 and adding 1 (if one writes "1" in the end). This series of numbers can be expressed by either of the two recurrent formulas:

$$I_0 = 0, \quad I_n = 2I_{n-1} + \left\{ \begin{array}{ll} 0, & \text{if } n \text{ is even,} \\ 1, & \text{if } n \text{ is odd,} \end{array} \right\} \tag{1}$$

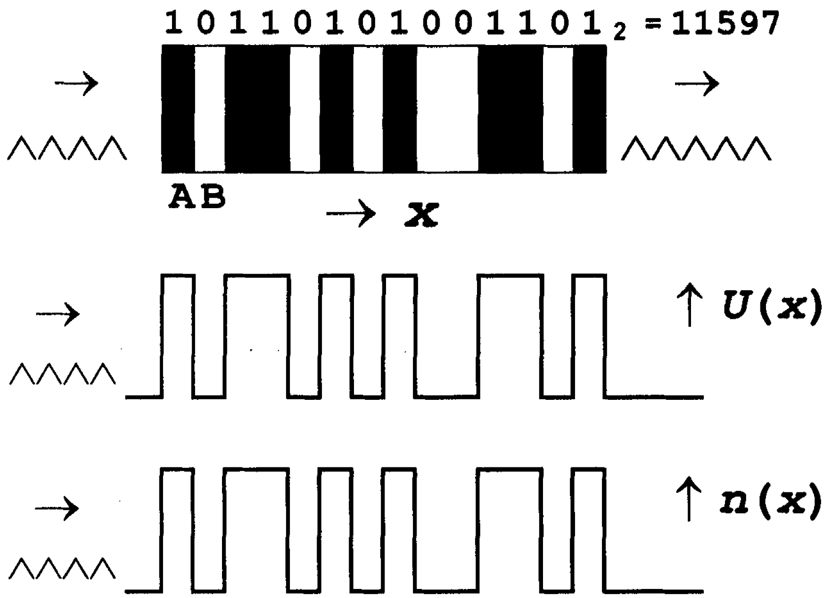

Fig. 1. A binary number, its representation in the form of a multilayer structure consisting of two substances, A and B, and the corresponding profiles of potential energy and refraction index.

$$I_0 = 0, \quad I_1 = 1, \quad I_n = 2^{n-1} + I_{n-2}. \tag{2}$$

In optics, structures of this type are known as multilayer dielectric mirrors whereas in nanoelectronics they are referred to as quantum well superlattices. Spectral portraits of these structures are presented in Fig. 2. A systematic sequential modification of spectra with increasing number "length" is clearly seen.

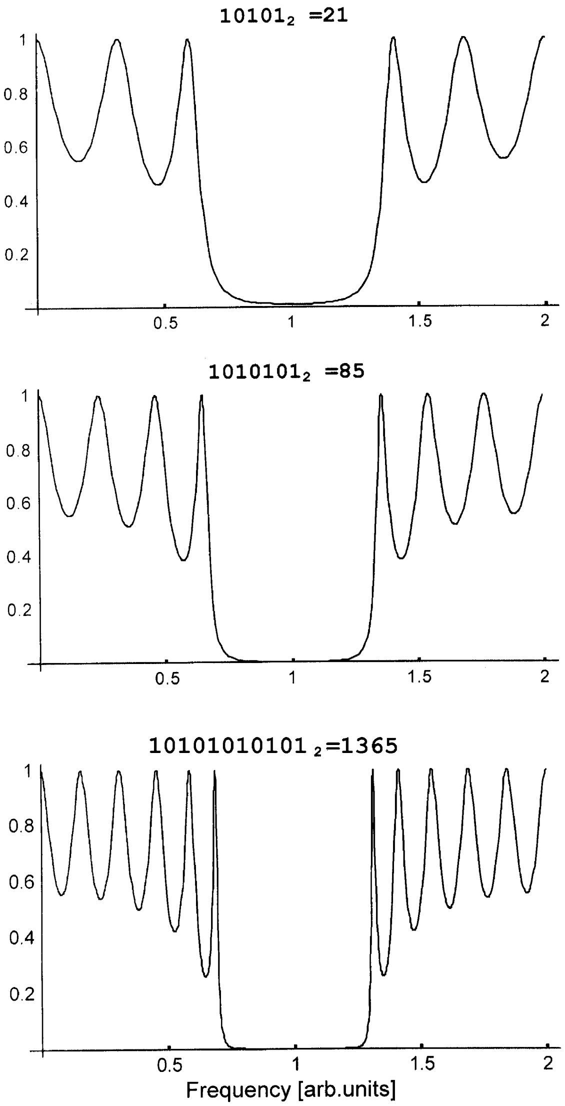

Fig. 2. Spectral portraits of binary numbers consisting of alternating unities and zeros. Values of such numbers are expressed by Eqs. (1) and (2).

Numbers which have several "0" and "1" in the middle and alternating "0" and "1" in the edges can be very well identified as well. These, e.g., are 101010000010101 or 101011110101. With respect to electromagnetic waves these structures form interferometers whose intermediate layer thickness corresponds to a set of similar digits (layers) in the center whereas alternating digits in the edges form mirrors. In this case, one or more sharp lines appear in transmission spectrum,the number and position of lines being determined by the number of digits in the central part of a binary number (Fig. 3). As we have mentioned above, development of resonant transmission bands in an optical interferometer is an analog of resonant tunneling in quantum structures. Note that the sides of the spectral portraits of a composite number show residual similarity to the proper original number (10101 in the case given in Fig. 3) but every transmission peak splits into two subbands. The splitting results from the symmetry of the structure. It occurs not only in optical structures but in symmetric quantum wells, in coupled identical pendulums, and in coupled LC-circuits.

Numbers with frequently alternating "0" and "1" are well recognized as well, e.g., 10010101 or 101101010 (Fig. 4). Replacing of one digit in a sequence of alternating "0" and "1" results in disappearance of one peak in the periodic spectrum and appearance of the new transmission band in the other part of the spectrum. Note, this modification corresponds to the appearance of a defect mode in a photonic crystal or an impurity state in energy spectrum of an electron in a crystal or superlattice. One can speak about donor or acceptor modes (states) depending on which digit, "0" or "1", is replaced by the complementary one.

Symmetry of transmission spectra with respect to wave propagation direction makes numbers non-distinguishable in a number of cases. This is valid for numbers differing in the order of digit positions, e.g., 110101 versus 101011, or 11110101 versus 10101111. Furthermore, zeros at the right hand cannot be recognized because zeros at the left hand are not meaningful. This means numbers expressed as $M \times 2^N$ can be identified only with

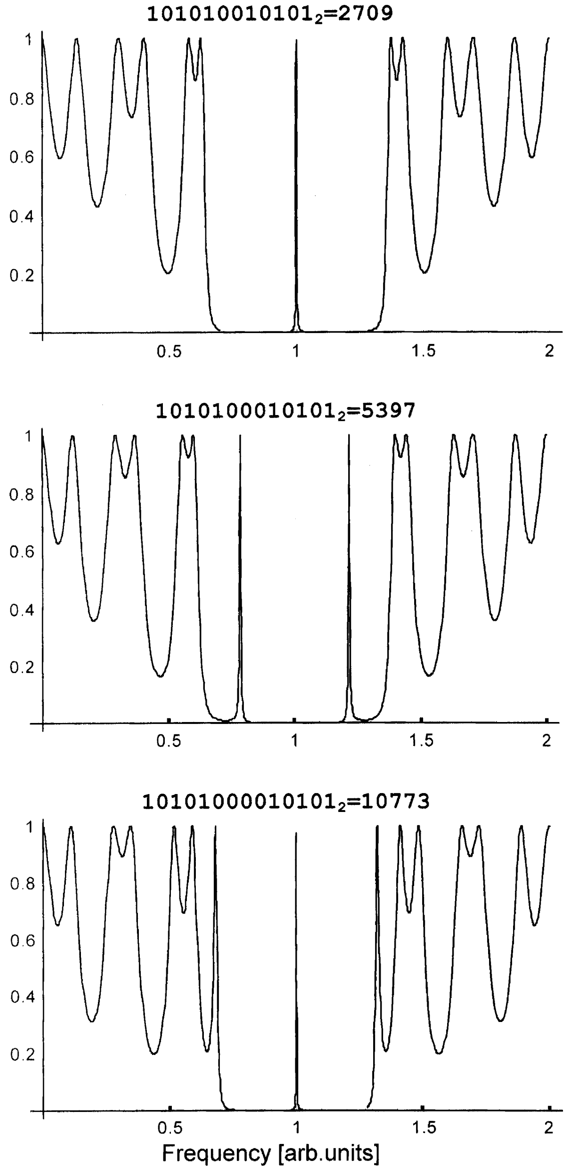

Fig. 3. The spectral portraits of numbers generated from two sequentially written $N = 10101_2 = 21$ numbers with 2, 3, and 4 zeros in the middle. A transition from one number to another one corresponds to operations described by Eq. (3).

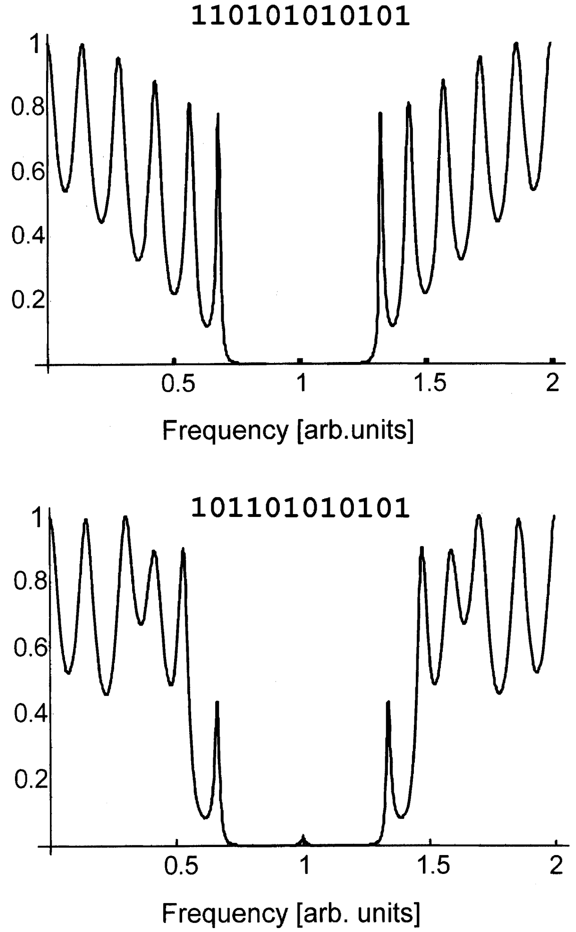

Fig. 4. Spectral portraits of two binary numbers differing in position of a single digit.

respect to mantissa M whereas the binary exponent N of the number remains unknown. In other words, for numbers like $M \times 2^N$ measurement of transmission spectrum provide reconstruction of the fractional part of the number logarithm but not the number value. However, recognition of two numbers differing in order of digit displacement as well as identification of numbers with several zeros at the end can be performed if a kind of asymmetry is introduced in the structure. This can be made, e.g., by varying layer parameters along wave propagation direction. Alternatively, at different edges of the structure different (with respect to refraction index) ambient environments can be created. For example, if a structure from one side is in contact with air ($n^* = 1$), a substrate can be placed from the other side with refraction index $n \neq n_0 \neq n_1 \neq n^*$. As a result, the "beginning" and the "end" of a multilayer number will become distinguishable.

4. Operations with numbers

Is it possible to perform operations with numbers using wave propagation through a sequence of multilayer numbers? Using structures of Fabry–Perot interferometer type we show that such a possibility *does* exist. An interferometer is a sequence of alternating layers broken in the middle. For example, if mirrors have the structure $101_2 = 5$ in the absence of an intermediate layers one has $101101_2 = 45$. In the general case, if the original mirror is identified as number N, two sequential mirrors can be expressed as number $M = N \times (2^k + 1)$, where k is the ceiling integer part of N on base 2, i.e., k is the number of digits in N written in base 2. Adding zeros in the middle of this structure gives a series of numbers

$$101101_2 = 45 = 5 \times 2^3 + 5,$$

$$1010101_2 = 85 = 5 \times 2^4 + 5,$$

$$10100101_2 = 165 = 5 \times 2^5 + 5$$

and so on. A systematic change in transmission spectrum of such a layer set upon expanding of an intermediate part is evident from Fig. 3. Here spectral portraits are given of three numbers resulting from two sequentially written numbers $N = 10101_2 = 21$ with two, three and four zeros in the middle. To summarize, a statement can be formulated: Sequential transmission of waves through a couple of numbers N spaced by the length $L = 0, 1, 2, \ldots$ corresponds to application of an operator to N

$$\hat{\mathbf{L}}(N) = N \times (2^{k+L} + 1), \tag{3}$$

where k is the number of digits in N written in base 2.

5. Information coding using full sets of numbers

In the previous sections it was shown that principally a spectral portrait can be assigned to every number. Is this assignment unique? In other words, is it always possible to reconstruct the original spatial distribution of a potential or a dielectric function using a given spectral portrait? Most probably, in general formulation the problem is not solvable. For quantum particles the analysis of inverse problems of the Schroedinger equation seems to be better examined [1] then analysis of inverse problems in the case of the Helmholtz equation. However even with respect to the Schroedinger equation the analysis is rather far from being complete. A problem can be formulated of identifying classes of sequences possessing a unique relationship between a spatial structure (i.e., a number value) and a resonant frequency (energy) spectrum relevant to non-reflective propagation of waves through such a structure. Numbers belonging to these classes feature unique spectral portraits and can be unambiguously recognized by means of wave propagation. In optics this property can be used in optical data recording and read-out, in nano- and opto-electronics it can be used in engineering nanostructures with predefined energy spectrum of electrons. It is clear that not only identification of a specific number but also the possibility to code and identify any given number is necessary for information coding. Most probably, an optimal solution of this problem can be gained by searching among all the sets allowing strict identification by spectral portraits for at least one *complete set* of numbers [2], i.e., a set providing representation of any given number as a sum of a few different numbers belonging to the set. In case such a set cannot be found, coding of numbers can be performed using representation of a given number as a sum or a product of numbers with unique portraits. However in this case the gain in information density will not be so significant as it might be in the case of a full set.

6. Spectral portraits versus factorization problem

In this section, we show that at least in certain cases analysis of spectral portraits provides hints to prime factors of numbers. Cantor numbers will be used as representative example. We consider, a binary number belongs to the class of Cantor numbers if position of unities and zeros in the number obey the law of Cantor sets. A *triadic* Cantor multilayer structure is generated by means of iterative substitution of a middle 1/3 part of a dielectric layer by a layer with different refraction

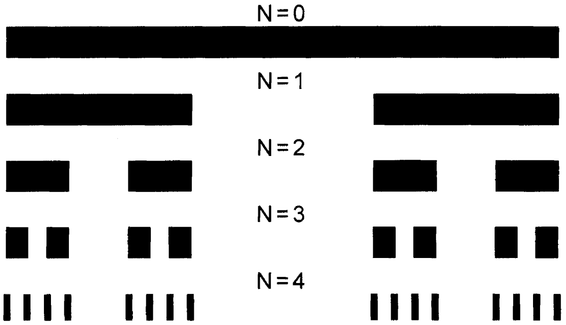

Fig. 5. Triadic Cantor structures. N is generation number.

index [3]. In Fig. 5 several junior generations of triadic Cantor structures are presented. In Table 1 a few junior generations of triadic Cantor numbers are written.

Generally, to build a Cantor lattice a linear segment is to be divided into odd number of equal parts, the even parts is then iteratively removed. These structures contain G^N layers, where G is generator equal to 3, 5, 7, ..., and $N = 1, 2, 3, \ldots$ is a generation number. Cantor structure comprises a fractal set with dimensionality

$$D(G) = \frac{\ln[(G+1)/2]}{\ln G}, \tag{4}$$

i.e., $D(3) = \ln 2/\ln 3$ for triadic (i.e., for $G = 3$) and $D(5) = \ln 3/\ln 5$ for pentadic (i.e., for $G = 5$) structures.

It is possible to build optical multilayer filters based on Cantor sets [4]. We performed a systematic analysis of spectral characteristics of optical Cantor filters [5] and revealed a pronounced correlation between spectral portraits of Cantor numbers and their prime factors.

A binary Cantor number value $C_2(G,N)$ of Nth generation with G-generator can be expressed by a general formula

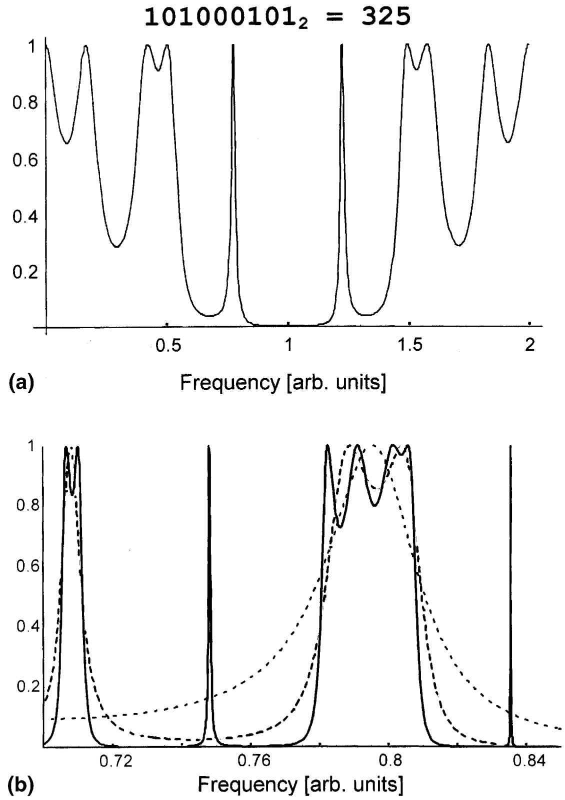

Fig. 6. (a) Spectral portrait of a triadic Cantor number of the second generation ($N = 2$) and (b) spectrum modification in the vicinity of a characteristic transmission peak for generations with $N = 2$ (short dash), 3 (long dash), and 4 (solid line).

$$C_2(G,N) = C_2(G,1) + \frac{1}{2}C_2(G,1)\sum_{i\neq j} 2^{|G^i - G^j|},$$
$$\text{with } i,j = 1,2,\ldots,G. \tag{5}$$

By means of systematic analysis of large number of Cantor structures with different generators, a sequential splitting of characteristic spectral lines was found to occur for successive generation number. The splitting is two-fold for triadic

Table 1
Triadic Cantor numbers

Generation	Binary code	Decimal value
$N = 1$	101	5
$N = 2$	101000101	$325 = 5 \times (2^0 + 2^6)$
$N = 3$	101000101000000000101000101	$85197125 = 5 \times (2^0 + 2^6 + 2^{18} + 2^{24})$

Table 2
Cantor binary numbers written in base 10 and their prime factors

Number of digits/layers	Decimal number	Prime factors
3	5	[**5**, 1]
9	325	[**5**, 2], [**13**, 1]
27	85197125	[**5**, 3], [**13**, 2], [**37**, 1], [**109**, 1]
81	1534774961612150361293125	[**5**, 4], [**13**, 3], [**37**, 2], [**109**, 2], [**246241**, 1], [**279073**, 1]
243	8972304477322525702813810177861539421333393918862058319149818714344653125	[**5**, 5], [**13**, 4], [**37**, 3], [**109**, 3], [**246241**, 2], [**279073**, 2] [**3618757**, 1], [**106979941**, 1], [**168410989**, 1], [**4977454861**, 1]

structures, three-fold for pentadic ones and $[(G+1)/2]$-fold for an arbitrary G value. The splitting gains a reasonable explanation in terms of coupled cavities by analogy with the above mentioned cases in classical and quantum mechanics and in electricity, i.e., coupled pendulums, quantum wells and LC-circuits [5]. In Fig. 6 a transmission spectrum of a triadic Cantor structure with $N = 2$ in the vicinity of a characteristic transmission peak for and its successive evolution $N = 2, 3, 4$ are shown. With increasing of generation number new peaks appear whereas the previously existing ones undergo double-splitting. The characteristic single peak for $N = 2$ at dimensionless frequency 0.795 splits into two peaks for $N = 3$ and into 4 peaks for $N = 4$. In its turn, the peak which is characteristic for $N = 3$ at frequency 0.710 splits into two peaks for $N = 4$.

Surprisingly, we found the *multiplets* of characteristic transmission peaks in spectral portraits of Cantor numbers does correlate with existence of *multiple prime factors* of the same numbers. In Table 2 decimal representations of triadic Cantor numbers with $N = 1, 2, 3, 4, 5$ and their prime factors are shown. The first number in square brackets is prime factor, whereas the second number in the brackets is its multiplicity. With increasing N, multiplicity of prime factors inherent in the previous generation increases by 1. Cantor numbers were found to possess definite sets of prime factors, every generator having its own unique set. For triadic numbers this is 5, 13, 37, 109,... For pentadic numbers this is 3, 7, 151, 331,...

The revealed correlation means spectral portraits not only provide a possibility to recognize numbers but also in certain cases contain instructive hints of number properties. In case of Cantor numbers, cor-relation of spectral multiplets with multiple prime factors makes a link between light propagation, number recognition, and number factorization.

7. Conclusions

The revealed correlations of geometrical structure and spectral portraits (transmission spectrum of electromagnetic waves, resonance tunneling spectrum of quantum particles) inherent in multilayer sets of different materials give a possibility to consider propagation of waves as number recognition. The approach can be utilized for number coding and data storage. It can be used in existing as well as new devices for data recording, storage and readout. These can be e.g., optical discs with multilayer coatings and optical readout, and multilayer nanostructures with tunnel microscope readout as well as other structures.

Note that energy dissipation is absent in the structures considered. In real structures dissipation can be minimized to make negligible effect on optical spectrum. Propagation of waves without energy dissipation results in complementary transmission and reflection spectra. Therefore, number recognition using multilayer structures can be performed not only in transmission mode but in reflection mode as well which might be beneficial in certain applications.

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